MATHEMATICS

Paper 9280/11
Paper 11

General comments

It is pleasing to record improvement in some of the areas mentioned in last year's report. For example, although there were still some candidates who divided each page into two columns, this practice was less prevalent than in previous years. If candidates need further reminding, they should work straight down the page rather than try to fit one question beside another question on the page.

General setting out was mostly satisfactory but there were a few questions where this was not the case. In **Question 3(i)**, for example, it was often the case that it was not at all clear which area was being considered. Answers to **Questions 4(i)**, **6** and **9(ii)** also suffered from poor or unclear setting out.

In previous reports comments were made that candidates were losing many marks on routine procedures. Although some improvement in this respect has been noted there is still room for further improvement.

Comments on specific questions

Question 1

Although most candidates attempted this question, often their attempts earned little or no credit. A large proportion of candidates did not recall that the standard method of showing that a given function is an increasing function is to consider the derivative of the function and to show that it is positive for all values of x. Large numbers of candidates did not differentiate but instead substituted a few particular values of x intending to show that f(x) increases as x increases. Unfortunately this is not a satisfactory method since it does not consider the general case. The candidates who attempted differentiation often made a mistake, either forgetting to multiply by the factor 2, or omitting completely the derivative of x, the second term. Finally, those candidates who obtained the correct derivative were often unsure what conclusion to draw. What was required was to state that 1 plus the square of any quantity is always positive and hence the function is increasing.

Answer: $f'(x) = 6(2x-5)^2 + 1$. This is > 0 for all values of x and hence the function is increasing.

Question 2

Most candidates started well and applied the binomial theorem for the first three terms. A few candidates confused the words 'ascending' with 'descending' and unfortunately this was a costly mistake. Much more common errors, however, were sign errors and not raising p to the power 2 when simplifying $15(-px)^2$. This last error was particularly costly since it prevented a quadratic equation being formed in part **(ii)**.

Answers: **(i)** $1-6px+15p^2x^2$; **(ii)** $-2/5$.

Question 3

Many candidates thought that the radius of the semicircle and the radius of the sector were the same. A particular feature of candidates' answers in this question was working in which it was not clear exactly what was being considered at each stage. For example words such as 'Area of sector = ...', 'Area of semicircle = ….' would have been helpful both to candidates themselves and to Examiners. The result for candidates was sometimes confused and incorrect work.

Answers: **(i)** $\pi/8$; **(ii)** $8+5\pi$.

Question 4

This question was generally very well done and full marks was frequently the outcome. In part **(i)** Exa[…] expected to see two equations, $ar^2 = -108$ and $ar^5 = 32$, and for candidates to proceed from there [by] eliminating a to find r. In reality, although many candidates did employ this method, it was also the case that many candidates employed rather more 'ad hoc' methods to reach the answer.

Answers: **(i)** $-2/3$; **(ii)** – 243; **(iii)** –145.8.

Question 5

Part **(i)** was usually done well. In part **(ii)**, most candidates reached a correct expression for $\sin^2\theta$ or $\cos^2\theta$ or $\tan^2\theta$ but very few candidates remembered the ± sign on taking the square root. Most candidates, therefore, only found two of the four solutions.

Answers: **(ii)** 54.7°, 125.3°, 234.7°, 305.3°.

Question 6

This question tended to expose a general lack of understanding and confidence in dealing with a number of processes involving vectors. Part **(i)** was done reasonably well with candidates required to demonstrate that the scalar product of **OA** and **OC** is zero. Part **(ii)**, however, was not so well done. It was disappointing to see many errors, even in the first step (finding **CA**) of this part. Candidates should have found that the **j** and **k** components were both zero and with this straightforward case the magnitude of **CA** is simply the coefficient of the **i** component. However, the majority of candidates who reached **CA** correctly did not recognise this and proceeded to square, add and square root etc., often not reaching the correct answer. In part **(iii)** many candidates made errors, including sign errors, in reaching **BA**. However, it was pleasing to see that, compared to previous years, a greater proportion of candidates were employing the correct method for finding a unit vector.

Answers: **(i)** **OA.OC** = $-4p^2 - q^2 + 4p^2 + q^2 = 0$ hence perpendicular; **(ii)** |**CA**| = $1 + 4p^2 + q^2$;
(iii) 1/9 (**i** + 4**j** +8**k**).

Question 7

Part **(i)** was generally answered well, although a common incorrect answer was (1½, 1½), obtained by subtracting instead of adding the end-points. Although part **(ii)** requires standard procedures few correct answers were seen. There is more than one way of tackling this question but the expected method is to equate the equation of the curve with the equation of the line, rearrange to make zero on one side and then apply the condition for equal roots ($b^2 - 4ac = 0$). Candidates who attempted this method were usually successful but other methods were far less successful.

Answers: **(i)** (2½, 2½); **(ii)** $m = -8$, (–2, 16).

Question 8

In part **(i)**, completing the square is a topic which appears almost every year and candidates are advised to practise this process so that they are confident that they can perform it accurately. In this particular case there were many correct answers but there was still a significant proportion of candidates who did not achieve all 3 marks. Parts **(ii)** and **(iii)** were answered reasonably well. Part **(iv)** was answered very well. Candidates were perhaps fortunate that the correct answer required the <u>positive</u> square root. There will be occasions, of course, when the answer will require the negative square root so the correct procedure is to apply the '±' sign in the first instance and then to decide which sign is appropriate.

Answers: **(i)** $2(x-3)^2 - 5$; **(ii)** 3; **(iii)** $y \geq 27$; **(iv)** $3 + \sqrt{\tfrac{1}{2}(x+5)}$ for $x \geq 27$.

Question 9

Most candidates used the suggested substitution and were able to transform the resulting equation term quadratic equation and solve it. Some candidates forgot to square the roots in order to find value In part **(ii)** the second derivative was usually found correctly. The expected method for determining nature of the stationary points was to substitute the values of x found in part **(i)** into the second derivative. Other valid methods are accepted but it is necessary to show the actual substitution of particular values of x. Part **(iii)** was usually done very well.

Answers: **(i)** $\frac{1}{9}$, 9; **(ii)** $f''(x) = \frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$, Maximum at $x = \frac{1}{9}$, Minimum at $x = 9$;
(iii) $f(x) = 2x^{\frac{3}{2}} + 6x^{\frac{1}{2}} - 10x + 5$.

Question 10

Parts **(i)** and **(ii)** were done very well and many candidates achieved full marks. Part **(iii)**, however, was far more challenging. Most candidates made the mistake of applying the same limits to the curve and the line when integrating. Other candidates ignored the tangent and simply integrated the equation of the curve between limits 5/4 and 2. Few candidates were able to score more than 1 mark for this part.

Answers: **(i)** $B(5/4,0)$, $C(0,3/4)$; **(ii)** $\sqrt{17}/4$; **(iii)** 3/40.

MATHEMATICS

Paper 9280/21
Paper 21

Key Messages

Candidates should read the questions carefully, making sure that they answer them fully. The appropriate degree of accuracy should be used and candidates should also check that they have their calculators in the correct mode for the question they are doing.

General Comments

A basic lack of understanding of logarithms appeared to be quite common among many candidates. This clearly affected their performance on both **Questions 1** and **2**.

Comments on Specific Questions

Question 1

Most candidates attempted to solve either the equation $(2^x - 7)^2 = 1$ or the equations $2^x - 7 = \pm 1$. While most candidates were able to obtain the solution $x = 3$, many had difficulty with the solution which resulted from $2^x = 6$ not recognising that the use of logarithms was required.

Answer: $x = 3$ and $x = 2.58$

Question 2

This question was probably the least well done on the paper. The basic laws of logarithms were seldom applied correctly, with the result that many candidates were unable to gain any marks. Few candidates were able to recognise that $2\ln x = \ln x^2$ and many candidates erroneously thought that $\ln(3 - 2x) = \ln 3 - \ln 2x$. The syllabus demands of the topic of logarithms is clearly an area that needs to be concentrated upon more.

For those candidates that were able to obtain the correct quadratic equation, it was pleasing to see that many gave or indicated that the positive solution was the only valid solution.

Answer: $x = 0.6$

Question 3

(i) Few candidates were able to show the required trigonometric relationship given by not considering double angles to start with. Many tried to make use of $\sin^2 x + \cos^2 x = 1$ which was of no real help. For those candidates that did try to use the double angle formulae, sign errors often prevented them from obtaining full marks. A few candidates started with $\frac{3}{2}(1 - \cos 4x)$ and were much more successful in obtaining $12\sin^2 x\cos^2 x$ as a result.

(ii) This part of the question was done by most candidates with a good deal more success. The given answer did help some candidates identify errors in their work which they were then able to correct. It should be noted that if a candidate fails to obtain a given answer and cannot see where they went wrong, it is far better to leave their work unaltered than to try to contrive to obtain the given answer by incorrect means. There are very often method marks available which candidates are otherwise unable to obtain if they use an incorrect method.

Question 4

This question was done very well by most candidates.

(i) Apart from those candidates who made simple algebraic or arithmetic errors, most were able to obtain the required values.

(ii) The required factorisation was usually done correctly using a variety of methods. Those candidates who had incorrect values from their work in part **(i)** were usually able to gain a method mark. It needs to be pointed out that those candidates who use synthetic division often ended up with a quadratic factor of $2x^2 - 8x + 6$ rather than $x^2 - 4x + 3$, but then had an extra factor of 2 when giving their final linear factors. Candidates should be encouraged to check that their linear factors are appropriate in such cases.

Answer: **(i)** $a = 2,\ b = -6$ **(ii)** $(x-1)(x-3)(2x+3)$

Question 5

This question was done very well by many candidates.

(i) Provided candidates recognised that they had to differentiate y as a product, most were able to gain full marks for this part. Again, the given answer was a help to candidates and acted as a prompt for those who did not readily recognise that they had to use the product rule.

(ii) Most candidates were able to make use of the given answer to find the appropriate gradient, together with the relevant coordinates and produce the equation of the normal.

Answer: **(ii)** $y = \frac{1}{2} - \frac{1}{2}x$

Question 6

(i) It was expected that candidates would make sketches of the graphs of $y = 4x - 2$ and $y = \cot x$ and show that there was one point of intersection in the given range. Most were able to produce a good sketch of $y = 4x - 2$ although it appeared that many did not make use of a ruler to draw a straight line. Few candidates were able to draw the graph of $y = \cot x$. When asked to produce a sketch of a graph, it is not necessary to use graph paper, but better to do the sketch within the body of the rest of the question solution.

(ii) Most candidates chose to adopt a change of sign method with great success as long as $\mathrm{f}(x) = \cot x - 4x + 2$ or equivalent was used. For those candidates who chose to substitute the given values into $\cot x = 4x - 2$, credit was only given if a fully correct explanation was given.

(iii) This part of the question was misunderstood by many candidates who seemed to think that some sort of numerical substitution was needed rather than just a re-writing of the equation $\cot x = 4x - 2$

(iv) The iteration process to obtain the root correct to 2 decimal places was usually done with success provided candidates had their calculator in the correct mode.

Answer: **(iv)** 0.76

Question 7

(i) A standard straightforward application of the syllabus which most candidates did really w from those who did not give the exact value of R but chose instead to give a value to 3 sign figures; candidates need to ensure that they understand what is meant by the phrase 'exact va of'.

(ii) Most solutions seen were calculated correctly with candidates performing the correct order of operations. Very few candidates however were able to obtain all the solutions in the given range, with most just giving the first 2 possible solutions.

(iii) This part of the question was 'the discriminator' for the paper. Very few correct solutions were seen as it required insight and deduction, using the maximum value the expression that had obtained in part **(i)**, but very little work, as indicated by the mark allocation.

Answer: **(i)** $R = \sqrt{29}$, $\theta = 21.80^{\circ}$, **(ii)** $13.3^{\circ}, 55.1^{\circ}, 193.2^{\circ}, 235.2^{\circ}$ **(iii)** $\frac{1}{116}$

MATHEMATICS

Paper 9280/41

Paper 41

General Comments

In the question paper each of the following is essential to the question,

- the motion of a car in **Question 2**,
- the motion of a distress signal in **Question 3**,
- the motion of a train in **Question 4**

and

- the motion of a car in **Question 7**.

Many candidates made sketches of pleasing images of these moving items, with waves on the sea below in the case of the distress signal. However sketches of this type do not help the candidate and can be a distraction.

The essence of the subject is that a real moving body is treated as a particle and the study of the particle's motion provides realistic information about the motion of the real body. Thus a diagram of worth to answering the question should consist of a tiny blob to represent the moving body, (or stationary body if relevant) with a straight line representing the surface of the sea or a road or a railway line and useful annotations such as a length, a weight, the magnitude of a force or the size of an angle.

In a considerable number of cases, the presentation of candidates' work was untidy. In some such cases candidates answered one question on the left hand side of a page and another question on the right hand side of the same page. In this circumstance the work of the two questions sometimes became intertwined. This procedure should be discouraged.

Candidates should read the questions very carefully. Unfortunately very many candidates did not do so in the case of **Question 6**.

Comments on Specific Questions

Question 1

It is unusual for a question to ask candidates for a statement, but the requirement of such was fairly well attempted in this question.

(i) Most candidates correctly wrote that the minimum vertical force required to move the block is less than the minimum horizontal force required to move the block, giving a coherent reason why this applied in the given circumstances.

(ii) In applying Newton's second law a fairly large minority of candidates disappointingly omitted the force of friction.

Question 2

In this question a car moves from the bottom to the top of a straight hill. In order to construct an equation allowing candidates to calculate the speed of the car when it reaches the top of the hill, candidates form a linear combination in which the work done by the driving force plus the decrease in the kinetic energy is equal to the increase in potential energy plus the work done against the resistance.

Candidates were aware of this strategy, but many omitted the increase in the potential en... candidates had a problem with units, with some terms in joules and others with kilojoules. Anoth... is to ensure correct signs, plus or minus, for all four terms.

Question 3

The first two parts of the question are very straightforward and candidates attempted them very well.

(i) All candidates recognised the scenario as that in which the speed had to be found of a particle being projected vertically upwards and reaching a height of 45 m above the point of projection. Candidates had no difficulty in finding the required speed.

(ii) The most common of several approaches to this question was for candidates to consider a particle released from rest and falling for 5m, and finding the time taken from $5 = \frac{1}{2} 10t^2$. The time is 1s and candidates allowed another 1s to accommodate the same time for the upward part as for the downward part.

Another method that many candidates used with complete satisfaction was to use the answer found in part **(i)**. The times at which the signal is at a height of 40 m are given by $40 = 30t - \frac{1}{2} 10t^2$. The two answers are 2s and 4s and correspond to passing the cliff top on the way up and on the way down. The interval from 2s to 4s is the time when the signal is above the cliff top.

(iii) This part was more testing than the first two, however most high scoring candidates scored all three of the marks available from this part of this question.

Question 4

Candidates recognised that the acceleration to be found in part **(i)** is instantaneous, whereas the speed to be found in **(ii)** relates to a period while the acceleration is zero and the speed is therefore constant. These contrasting scenarios caused no problem for the majority of candidates.

(i) Almost all candidates seemed to be aware of finding the driving force by dividing the power by the speed and using this in the application of Newton's second law to find the required acceleration.

(ii) Almost all candidates interpreted 'steady' correctly, as constant speed. Again candidates were aware of finding the driving force by dividing the power by the speed, with again using it in the application of Newton's second law but this time to find the constant (steady) speed as required.

Question 5

Some candidates may have found it daunting that the first requirement, of a connected particles question, be to find the normal and frictional components of a contact force. However the question was fairly well attempted.

(i) As usual with connected particles questions candidates applied Newton's second law to each of the particles, facilitating simultaneous equations in tension and acceleration. Candidates coped with this very well.

(ii) Candidates found no difficulty to find the required distance.

Question 6

The question says that the particle lies on a horizontal plane and is subject to horizontal forces. T be clearly understood by candidates, but nevertheless a considerable number of candidates had the w of the particle acting along the negative *y*-axis, which is clearly in the horizontal plane.

(i) Almost all candidates found Fcosθ and Fsinθ by equating them to the sum of the *x*-components and the sum of the *y*-components, respectively, of the other horizontal forces. Unfortunately a large minority included the weight with the *y*-components.

(ii) Most candidates realised that the resultant of the three (not four) remaining forces has magnitude equal to that of the removed force, and direction opposite to that of the direction of the removed force. Most candidates appropriately used F = 0.5a to obtain the required acceleration.

Question 7

Candidates seemed very familiar with the chain s(t) ➔ v(t) ➔ a(t) by differentiation, and the reverse by integration.

(i) Candidates seemed hesitant to find v(t) because the first requirement is to find a distance, for which the s(t) function of t is available. However most candidates realised the need to find v(t) and to solve v(t) = 0 to find the time at which the car is at B. Candidates who achieved this had no difficulty with substituting the value of t into s(t) to obtain the required answer.

(ii) This part requires the maximum speed to be found. Candidates were clearly aware that solving dv/dt = 0 was the starting point, to obtain the value of t at which the maximum speed occurs. Candidates who reached this far were aware that substitution into v(t) is all that is required.

(iii) Most candidates realised the need to evaluate a(0) and a(100) to obtain the required answers.

(iv) The attempts at a sketch of the velocity-time graph were generally disappointing. Most candidates showed the starting point and ending point at (0, 0) and (100, 0) as expected. Most candidates identified (66.7, 20.8) as the maximum point, but the sketch of many candidates consisted of straight line segments from (0, 0) to (66.7, 20.8) and from (66.7, 20.8) to (100, 0).

Very few candidates demonstrated the zero acceleration at the origin with the curve having the t-axis as a tangent. Furthermore few had the slope of the curve continuously increasing from zero at the origin to a point where the slope starts continuously decreasing until it become zero at the maximum velocity point.